Experiment 4 (M6)

The Projectile Motion

1- Purpose

To examine the projectile motion in the inclined plane. To find the time of flight, horizontal range and maximum height of a projectile for different velocity, angle of projection.

2- Apparatus

Air table set, launcher, wooden block, ruler.

3- Theory

Projectile is defined as, any body thrown with some initial velocity, which is then allowed to move under the action of gravity alone. For a given initial velocity, $\mathbf{v_o}$, and initial position, $\mathbf{s_o}$, the position of a particle, $\mathbf{s}$, as a function of time, undergoing constant acceleration, $\mathbf{a}$ is given by

$$\boldsymbol{s} = \boldsymbol{s_o} + \boldsymbol{v_o} t + \frac{1}{2}\boldsymbol{a} t^2 \quad (1)$$

This is a vector equation and can be broken up into its x, y, and z components. Since the motion is in a plane, we need only look at the x and y components. If we neglect air resistance, the acceleration in the y direction is $-\boldsymbol{g}$, due to gravity. The acceleration in the x direction is zero. Hence, the vector equation (1) becomes two scalar equations:

$$\boldsymbol{x} = \boldsymbol{x_o} + \boldsymbol{v_o} t \quad (2)$$

$$\boldsymbol{y} = \boldsymbol{y_o} + \boldsymbol{v_o} t - \frac{1}{2}\boldsymbol{g} t^2 \quad (3)$$

In terms of the angle θ, and the initial speed vo, the initial velocity components are

$$v_{o_x} = v_o cos\theta \quad and \; v_{o_y} = v_o sin\theta \tag{4}$$

At any *t* time, velocity and magnitude of this velocity,

$$v_x = |\boldsymbol{v_o}|cos\theta \;\; ; \;\; v_y = |\boldsymbol{v_o}|sin\theta \;\; ; \; |v| = \left({v_x}^2 + {v_y}^2\right)^{1/2} \tag{5}$$

Due to there is no any acting net force on to the disc along to x-axis, the accelaration is zero ($a_x = 0$). On the other hand the gravity force effects to the disc motion along to y-axis;

$$\boldsymbol{F_y} = m\boldsymbol{a_y} \, ; \;\; \boldsymbol{a_y} = -\boldsymbol{g} sin\emptyset \tag{6}$$

Therefore the motion of the disc (x and y axis) can be defined with these equations which are shown below (Figure 1)

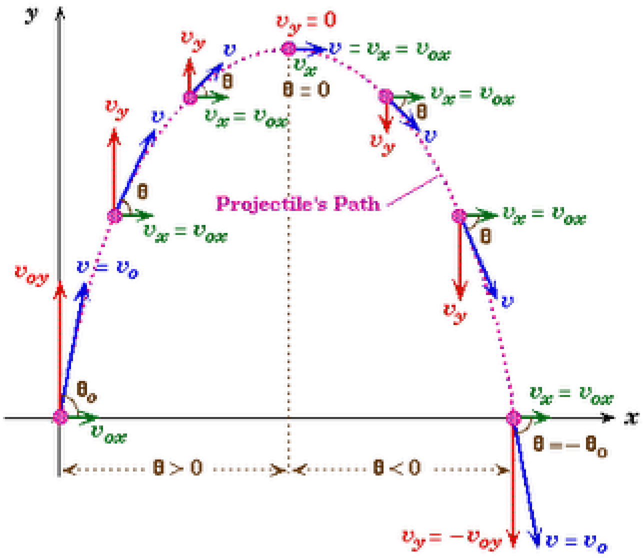

Figure 1 : The trajectory of the projectile motion in the inclined plane

The velocity and motion in the horizontal component of the disc has an equation as;

$$\boldsymbol{v_x} = \boldsymbol{v_{o_x}} = \boldsymbol{v_o} cos\theta = constant \tag{7}$$

$$\boldsymbol{x} = \boldsymbol{v_{o_x}} t = (\boldsymbol{v_o} cos\theta)t \tag{8}$$

On the other hand , the velocity and motion in the vertical component of the disc has an equation as;

$$\boldsymbol{v_y} = \boldsymbol{v_{o_y}} - \boldsymbol{a}t = \boldsymbol{v_o} sin\theta - \boldsymbol{a}t \tag{9}$$

$$\boldsymbol{y} = \boldsymbol{v_{o_y}} t \; - \frac{1}{2}\boldsymbol{a}t^2 = (\boldsymbol{v_o} sin\theta)t - \frac{1}{2}\boldsymbol{a}t^2 \tag{10}$$

If we solve Eq. 8 for ***t*** and substitute the result into Eq. 10, then we get;

$$y = (tan\theta)x - (a/2v_o^2 cos\theta^2)x^2 \tag{11}$$

This Eq.11 describes that the motion of disc is parabolic. t^* time takes when the disc reach maximum point of this trajectory (parobol) after release from the ground (bottom of air table), and at this point the velocity in the vertical compenent ($v_y = 0$) is zero

$$\boldsymbol{v_y} = \boldsymbol{v_o} sin\theta - \boldsymbol{a}t^* = 0 \tag{12}$$

From Eq.12 , the rising time is (t^*)

$$t^* = t_{rising} = \frac{v_o sin\theta}{a} \tag{13}$$

When you rewrite t^* instead of t in Eq. 10, then we can get (maximum height);

$$y = h = (v_o^2 sin^2\theta)/2a \tag{14}$$

And then, Flight time takes $2t^*$, so;

$$t_{fly} = 2t^* = \frac{2v_o sin\theta}{a} \tag{15}$$

The range in an horizantal axis (travelled distance on x - axis), R ;

$$x = R = (v_o cos\theta)t_{fly} = (v_o cos\theta)\frac{2v_o sin\theta}{a} \tag{16}$$

And using $\sin 2\theta = 2\sin\theta\cos\theta$, we can rewrite Eq.16;

$$R = \frac{v_o^2 sin2\theta}{a} \tag{17}$$

When the angle is $\theta = 45^\circ$, The disc reachs maximum range in an horizantal axis. +1 is the maximum value of the $sin2\theta$. So when the angle is changed , the range (R) is changed ! (Figure 2)

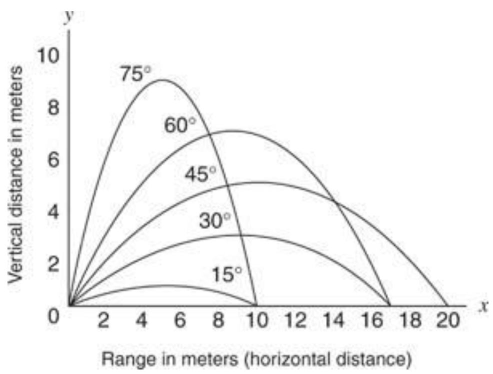

Figure 2: Different angle and the same velocity

4- Procedure

1. Operate air compressor and balance the air table.
2. Locate the wooden block to the back leg of the air table and find the slope of the air table.
3. Keep one of the discs (pucks) unmoved at the left (or right) upper side of the air table under the data sheet.
4. Locate the launcher, whose angle is adjusted to 45^o, at the right (or left) lower side of the air table.
5. Put the other disc (puck) to the launcher and practise launching. When you are ready, follow 6^{th} step.
6. Adjust spark timer (generator) to a proper frequency or period.
7. Operate again air compressor and launch the disc (puck). Be careful while gethering data (path A): As soon as you launch the disc (puck) push the spark timer (generator) pedal.
8. Just after you gether horizontal projectile motion data, locate the disc (puck) to the top of the air table and gether the free fall data (path B).
9. Check out your data if it is proper.
10. Give numbers to your data points 0,1,2… starting from first data point for the projictile motion.
11. Find out x- and y- axes on your data sheet (apply parallel fitting).
12. Find flight time (t_{fly}), rising time(t_{rising}) , and check whether $2t_{rising}$= t_{fly} or not. If not , Why?
13. Find the range (travelled distance on x – axis (R)),
14. Find the v_{o_x} , v_{o_y} and v_o
15. Find the maximum height, ($h_{max_{exp}}$)
16. Find the maximum height using Eq.14 ($h_{max_{teo}}$)
17. And check whether $h_{max_{exp}}$= $h_{max_{teo}}$or not. If not , Why?

5- Questions

1. Analyse the motion in x- and y- axes. Which type of motion does the disc follow?